

## 1.4 Continuity and the IVT

Obj: Find intervals of continuity; Describe types of discontinuity; Understand and use the Intermediate Value theorem

**Mathematical definition of continuity has 3 parts.**

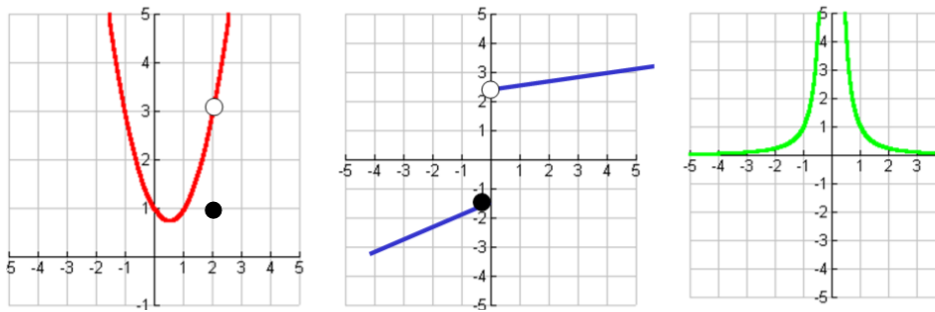
A function is continuous at  $x=c$  if:

1.

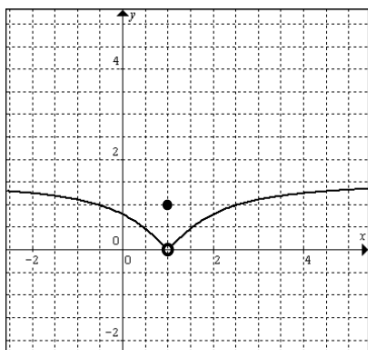
2.

3.

There are 3 types of discontinuity.



Explain why these are discontinuous using the definition.



$$\lim_{x \rightarrow 1^-} f(x)$$

$$\lim_{x \rightarrow 1^+} f(x)$$

$$\lim_{x \rightarrow 1} f(x)$$

$$f(1) =$$

Is the function continuous? Use your definition to justify.

Removable discontinuities. What causes a removable discontinuity?

$$y = \frac{x-1}{x^2 - 4x + 3}$$

Two common sense theorems. Draw examples.

$f(x)$  is continuous on the open interval  $(a,b)$  iff  $f(x)$  is continuous at every point in the interval.

$f(x)$  is continuous on the closed interval  $[a,b]$  iff it is continuous on  $(a,b)$  and continuous from the right at  $a$  and continuous from the left at  $b$ .

For what interval is  $y = \frac{x-1}{x^2 - 4x + 3}$  Continuous?

Is  $f(x)$  continuous along the closed interval of  $[-1, 1]$

Justify your answer with the definition.

A very common AP MC problem.

For what value  $k$  is the function continuous?  $f(x) = \begin{cases} \frac{x^2 - 144}{x - 12} & x \neq 12 \\ k, & x = 12 \end{cases}$

Find  $a$  and  $b$  so that  $f$  is everywhere continuous.

$$f(x) = \begin{cases} 2, & x \leq 1 \\ ax + b, & -1 < x < 3 \\ -2, & x \geq 3 \end{cases}$$

More common sense theorems:

**Intermediate Value Theorem. "IVT" One of the most used on the AP test!**

Given a continuous function on a closed interval  $[a, b]$  then that function takes on every value between  $f(a)$  and  $f(b)$

Draw a picture!

Use the IVT to show that  $f(x) = x^2 + x - 6$  has a root in the interval  $[1, 3]$ . Then find the root.

**Justification.** Since  $f(x)$  is continuous, and  $f(\quad) = \quad$  and  $f(\quad) = \quad$ , by the IVT, there exists a  $c$  in  $[\quad]$ , such that  $f(c) = \quad$  (or within  $[\quad]$ )

You try. Apply the IVT, if possible, on  $[0,5]$  so that  $f(c)=11$  for the function  $f(x)=x^2+x-1$

time	0	5	7	9	14
V(t)	3.35	4.25	2.75	2.55	4.70

You try. The table above gives several measurements of the velocity of a particle moving along a straight line. What is the smallest possible number of times where  $v(t)$  is exactly 4 m/s if  $v(t)$  is a continuous function?

- a. 0                      b. 1                      c. 2                      d. 3                      e. 4